Unit Circle
Trigonometry:

is the study of three sided figures or triangles. It analyzes the relationships between the lengths of their sides and the measure of its angles.

Trigonometry was established from a need to locate points and to measure distances on land and in space.
Applications of trigonometry

What can you do with trig?

Historically, it was developed for astronomy and geography, but scientists have been using it for centuries for other purposes, too.

Besides other fields of mathematics, trig is used in physics, engineering, and chemistry.

Within mathematics, trig is used primarily in calculus (which is perhaps its greatest application), linear algebra, and statistics. Since these fields are used throughout the natural and social sciences, trig is a very useful subject to know.
Astronomy and geography

Trigonometric tables were created over two thousand years ago for computations in astronomy. The stars were thought to be fixed on a crystal sphere of great size, and that model was perfect for practical purposes. Only the planets moved on the sphere.

(At the time there were seven recognized planets: Mercury, Venus, Mars, Jupiter, Saturn, the moon, and the sun. Those are the planets that we name our days of the week after. The earth wasn't yet considered to be a planet since it was the center of the universe, and the outer planets weren't discovered then.)
The kind of trigonometry needed to understand positions on a sphere is called spherical trigonometry. Spherical trigonometry is rarely taught now since its job has been taken over by linear algebra. Nonetheless, one application of trigonometry is astronomy.
Real Life

It will be in Calculus A LOT because the derivative in trig’s are simple.

But if you plan on constructing or working with spherical shapes, you will apply the unit circle.

Sailors, Navy/Army/Airforce/Coast Guard use “unit circle” to find locations.

1 degree = 60 nautical miles (3,600 square miles)
The Unit Circle:

is a tool used in understanding sines and cosines of angles found in right triangles. Its radius is exactly one unit in length, usually just called "one".

The circle's center is at the origin, and its circumference comprises the set of all points that are exactly one unit from the origin while lying in the plane.

The circle has its own formula:

\[ x^2 + y^2 = r^2 \]
The radius equals one unit in the unit circle, and the center of the circle is the origin. Can you name 4 points on the unit circle?
To make an angle on the unit circle:

There are two sides of an angle.

**Initial side** of an angle - is a fixed ray along the x--axis.

**Terminal side** of an angle - is a ray that does the moving in either a counterclockwise direction if the angle is positive or clockwise if the angle is negative.

An angle is in **standard position** if the vertex is the origin and the initial side is on the positive side of the x--axis.

**Quadrantal Angle** in standard position - has its terminal side on either the x or y-axis.
The Greek Letter Theta θ

θ is a Greek letter that we use as a variable to represent angles in Trigonometry.

It can be a degree measure (DEG) or it can be a radian measure (RAD).

Positive angles open counterclockwise and negative angles open clockwise

\[ \cos(\theta) = \text{X-value on the unit circle} \]
\[ \sin(\theta) = \text{Y-value on the unit circle} \]
\[ \tan(\theta) = \frac{Y}{X} \text{ or } \frac{\sin(\theta)}{\cos(\theta)} \]
Why do we use Radians?

1. Radians have evolved from a circle.
2. A radian is a dimensionless quantity
3. Radians provide simple formulae in terms of derivations of arc lengths and sector areas.

A radian is the ratio of an arc length to a radius.

\[ S = r \theta \]

Arc length \( S \) = \( r \theta \)  
\( r \) = radius 
\( \theta \) = angle measure in radians

If not in radians convert \( \frac{\pi}{180} \)
Trig in a family of 6

Cosecant is the reciprocal of Sine.
Secant is the reciprocal of Cosine.
Cotangent is the reciprocal of Tangent.

Reciprocal in everyday language is to “flip”.

Inverse is to switch the values of x & y (we use inverse to find angle measures).
Right Triangle Trig to Unit Circle Trig

\[
\begin{align*}
\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
\tan \theta &= \frac{\text{opposite}}{\text{adjacent}} \\
\csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\
\sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\
\cot \theta &= \frac{\text{adjacent}}{\text{opposite}}
\end{align*}
\]

Circle \( \Rightarrow x^2 + y^2 = r^2 \)

Terminal Side of \( \theta \) use \( r = \text{radius} \)

\[
\begin{align*}
\sin \theta &= \frac{y}{r} \\
\cos \theta &= \frac{x}{r} \\
\tan \theta &= \frac{y}{x} \\
\csc \theta &= \frac{r}{y} \\
\sec \theta &= \frac{r}{x} \\
\cot \theta &= \frac{x}{y}
\end{align*}
\]